Curricula for Mathematics: A comparative study of the United States, China, and International Baccalaureate

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Abstract
This study offers a comprehensive textual analysis comparing the mathematical curriculum objectives across the International Baccalaureate Diploma Programme (IBDP), China's Ordinary High School Mathematics Curriculum Standards, and the Common Core State Standards Initiative. Using an analytic comparison methodology, the mathematical knowledge and competencies imparted by each curriculum were measured and contrasted. The findings reveal that, when advanced placement courses are excluded, students in China are exposed to the highest number of mathematical knowledge points. However, when including advanced placement courses, the Common Core marginally surpasses the others in the density of mathematical content. Notably, across all three curricula, there's a limited emphasis on technological applications, leading to a stronger focus on computational mathematics and scientific calculations. Methods such as variation and optimization are scarcely touched upon in these curricula.

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Introduction and background

The International Baccalaureate Organization (IBO) is a renowned non-profit educational institution, affiliated with UNESCO, dedicated to the holistic development of students. Its secondary education program, commonly known as the IB, offers two distinct mathematical tracks: 'Analysis and Approaches' (AA) and 'Applications and Interpretation' (AI). These courses are internationally acclaimed and accepted by myriad universities in over 100 countries and regions globally (Xu, H., 2015).

The "Ordinary High School Mathematics Curriculum Standards" serves as an authoritative guideline issued by China's Ministry of Education. It aims to harmonize content and pedagogical requirements for high school mathematics, offering both educators and students a cohesive instructional framework. China's foundational education ranks among the world's elite. This is further exemplified by recent PISA results (e.g., OECD 2019) where Shanghai's students

showcased exemplary performance, drawing attention and sparking discussions among global educational scholars, policymakers, and practitioners (Fan, L., et al., 2021).

In the United States, the considerable autonomy granted to individual states has led to variations in the standard of mathematics education across the country. Despite this, some states have still managed to achieve impressive results in the PISA assessments (Alan H., S., & Jeremy, K., 2013). Recent evidence indicates a transformation in American classrooms following the introduction of the Common Core (CC) standards (Amy, R. et al., 2016). Furthermore, the Common Core curriculum aligns more scientifically with the TIMSS, thereby potentially improving student achievement (William H., S., & Richard T., H., 2012).

Given the robust professional foundations and demonstrated successes of these three curriculums, it is essential to undertake a deeper examination of their features and implications.

Research Questions

RQ1: How many mathematical knowledge points do gifted mathematics students recognize in the course compared to those students enrolled in IB, MC, and CC programs?

RQ2: To what extent can these knowledge points be incorporated into the broader realm of mathematics?

RQ3: What are the priorities and trade-offs inherent to the IB, MC, and CC when focusing on the acquisition of knowledge points?

To ground our research in a clear framework, the following foundational assumptions should be admitted:

AM1: Gifted students are inclined to select advanced mathematics courses over their foundational counterparts.

AM2: Gifted students possess the capability to master all knowledge points presented in these advanced courses.

The aspiration in addressing these queries is twofold: firstly, to delineate the variations in knowledge point allocation among the three curricula, and secondly, to furnish empirical evidence that might bridge current research lacunae. Concurrently, we anticipate offering insights beneficial to curriculum design and structuring, especially from the vantage point of knowledge volume.

Literature Review

Overview of Current Research on Curriculum Design

From the aforementioned details, the three curricula appear robustly designed, seemingly fostering the development of a vast number of competent individuals. Given this, it's not unexpected that scholars have investigated the factors contributing to these outcomes. Research has delved into various facets, including classroom and teaching dynamics (Boylan, M. et al., 2018; Wu, Y., & Bao, J., 2016), cultural and policy influences (Tan, C., 2013), and foundational backdrops (Zhang, M., & Kong, L. 2012). While extensive studies encompass many aspects of mathematics education, curiously, a limited number have centered on the curriculum itself. A keyword query on the Crossref* database revealed a mere 73 publications, each with more than 10 citations, that exclusively focus on the mathematics curriculum. In contrast, numerous articles adopt curriculum standards merely as guidelines or referential content, directing their primary analysis towards students (e.g., Jeannie C, H., Karen, N., 1999; Douglas H., C., and Julie S., 2013), teachers (e.g., Gwendolyn M., L., and Melvin, W. 1998; Pepin, B., et al, 2017), or textbooks (e.g., Tom, M., and Sheila, H., 2010; Fan, L., et al., 2021).

In the realm of curriculum studies, research appears to predominantly adopt a holistic or generalized approach. S.E. Anderson, in his article "Fighting Eurocentrism in Mathematics"

(Anderson, S.E., 1990), alluded to such tendencies. Another provocative assertion suggests that most of the world's curricula are mere adaptations of one of six foundational systems: Prussian, Russian, French, English, Japanese, and American (Cummings, 1999). In essence, there seems to be a scarcity of research that delves deeply into the specific content of curricula. This is particularly evident in the domain of mathematics, where there's an apparent lack of focus on individual knowledge points within the syllabus.

Remarkably, most existing studies addressing the curriculum hail from the latter part of the last century, with discussions specific to the curriculum itself seeming to wane in recent years. Given this context, it becomes imperative to examine 21st-century syllabi to assess their adequacy in meeting contemporary requirements.

Theoretical Framework

Through textual analysis, our intention is to delve into three curricula that have been extensively endorsed by a multitude of education experts and researchers. Our aim is to scrutinize widely accepted educational paradigms and evaluate the veracity of these assertions. To facilitate this, we've adopted the logical method of comparison, specifically, the analytical comparison—a technique championed by the 19th-century philosopher J.S. Mill (Feng, X., 2021).

Comparative analysis can be bifurcated into two main methodologies: the method of agreement and the method of difference. Employing the method of agreement, we will explore the congruencies in knowledge points among the triad of curriculum outlines, interpreting the logic behind such arrangements through a mathematical lens (Feng, X., 2021). Conversely, the method of difference will guide the exploration of variances in knowledge points, shedding light on the origins and implications of these discrepancies.

The specific research process is carried out according to the following process in Figure 1.
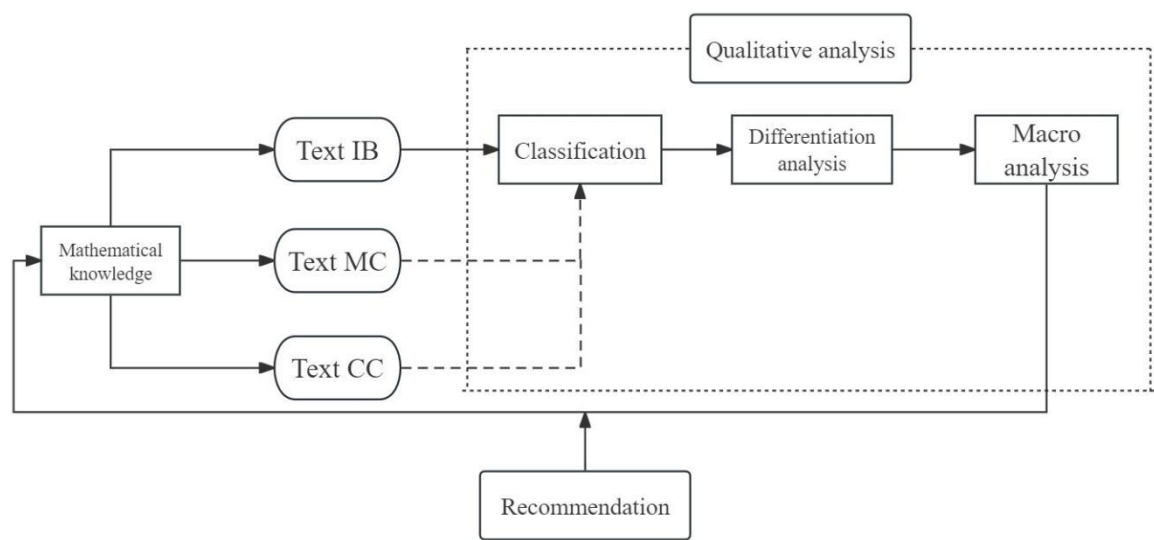

Figure 1
Research flowchart

Methodology

Procedures and materials

The classification standard used is the IB curriculum, which is the most commonly used in countries and regions internationally, as the benchmark curriculum standard. CC and MC are used as controls. The detailed information of these three curriculum standards is shown in Table 1 below.

Table 1

Specific information on the three curricula

Publishing entity	File name	Year of publication
International Baccalaureate Organization	Mathematics: analysis and approaches guide	2019
International Baccalaureate Organization	Mathematics: applications and interpretation guide	2019
China's Ministry of Education	Ordinary High School Mathematics Curriculum Standards	2020
National Council of Teachers of Mathematics	Common Core State Standards for Mathematics	2010

It is worth noting that the IB curriculum standards were first applied to testing in 2021, and MC was first applied to testing in 2022. However, CC was relatively early and put into testing around 2012, and CC has been in use without further modifications. So these three course standards are currently being used, although there are pros and cons, it does not affect the comparison of expected courses.

The curriculum of IB

Course content is bifurcated into two distinct categories: Analysis and Approaches (AA) and Applications and Interpretation (AI). The AA track underscores the growing reliance of global innovation on a profound understanding of mathematics, emphasizing the importance of analyzing specialized knowledge. Conversely, AI emphasizes the increasing symbiosis between mathematics and technology in an ever-evolving, data-driven world (IB Diploma Programme, 2019).

While these two tracks operate independently, students are only required to select one pathway to fulfill their mathematical studies. Given variations in students' mathematical aptitudes and examination complexities, both AA and AI offer two tiers of difficulty: Standard Level (SL) followed by High Level (HL) for those seeking a more advanced curriculum. Each track encompasses five core knowledge areas: number and algebra, functions, geometry and trigonometry, statistics and probability, and calculus. This structure forms the foundation for our comparative analysis based on topic.

For the purposes of this study, our focus is solely on mathematically gifted students; thus, we default to examining content specific to the High-Level (HL) mathematics curriculum.

Ordinary High School Mathematics Curriculum Standards

The *Ordinary High School Mathematics Curriculum Standards* (MC) is an authoritative document issued by the Ministry of Education of the People's Republic of China, outlining the instructional framework for senior high school mathematics, introduced in 2017. It underwent revisions in 2020, culminating in its current comprehensive form. The curriculum delineates three primary course categories: compulsory, semi-compulsory, and elective. These elective courses are categorized from Class A through E, tailored according to students' mathematical competencies and their projected future professions.

Class A is structured for students with a strong inclination towards disciplines like mathematics, physics, and computer science. Conversely, Class B is designed for those envisioning careers in fields such as economics, mathematical economics, sociology, chemistry, biology, and engineering. Class C is aimed at students with a humanities focus, Class D caters to those inclined towards arts, sports, and related fields, and Class E serves as a broader category without stringent specifications at present (Shi and Wang, 2019).

In parallel with our approach to the IB system, our analysis will predominantly focus on the courses with the most rigorous mathematical expectations, specifically, Class A and Class B.

Common Core State Standards for Mathematics

Initiated in July 2009, the *Common Core State Standards for Mathematics* (CC) was established on a foundation of comparative studies between the United States and high-performing nations on international tests. It also draws insights from the National Mathematics Commission's survey on students' mathematical proficiency, studies assessing self-directed learning capabilities, and other pivotal research, especially those focusing on the modalities of students' mathematical learning. The formulation and evolution of the CC received endorsement from 48 states, two territories, and Washington, D.C. The process was not only enriched by the contributions of teachers nationwide but also benefited from invaluable recommendations from organizations like the National Education Association and the American Teachers' Union.

A distinctive feature of the CC is its non-hierarchical organization of content. Instead of categorizing by difficulty levels, content is consolidated into Categories Ⅰ, Ⅱ, and Ⅲ. Notably, certain knowledge points don't align strictly within these three categories. For the sake of research clarity, we have encompassed all knowledge points, assuming that students can master those not explicitly mentioned in the three categories.

Worth noting is the U.S. education system's emphasis on students' autonomous choices. Exceptionally gifted students have the option to undertake college-level courses in advance, such as the Advanced Placement (AP) courses. Within this curriculum, mathematics is explored through two primary subjects: calculus and statistics. While there isn't a rigidly prescribed standard for this segment, considering the high enrollment of mathematically gifted students, we've generated an informal knowledge point code using popular reference texts like Barron's AP (David, B., Dennis, D., Shirley O, H., 2018; Martin, S., 2018). This discourse complements our comparisons of the IB, MC, and CC curricula.

Classification and Coding

The mathematical teaching contents in the syllabuses of IB, MC, and CC are independently coded using the axial coding, and the mathematical knowledge points involved in the three syllabuses are classified according to five major themes, namely, number and algebra, function, geometry and triangle, statistics and probability, and calculus which is based on the IB curriculum. On the basis of these five primary categories, 92 second level categories are divided. The specific coding principles and categories are shown in Table 2 below. Due to the complexity of the specific content, it will not be elaborated on one by one here. Instead, the number of third level classifications under each second level classification will be marked, which is the reference point.

Table 2

Coding category system based on mathematical knowledge points

First level category	Second level category	Category description	Reference Point
Number and Algebra	Set	The Concept and representation of Sets	3
	Equations and Inequalities	Properties of equations and inequalities	2
	Common logical terms	Necessary and sufficient conditions	4
Functions	Series	Arithmetic sequence, geometric series, etc.	4
	Concept and properties	Concept and properties of functions	3

	of functions		
	Power function, exponential function, logarithmic function	Power function, exponential function, logarithmic function	9
	Trigonometric function	Trigonometric identity transformation, concept and properties of trigonometric functions, etc.	5
Geometry and trigonometry	Plane vectors and their applications	Basic theorem of vector and coordinate representation, vector applications and solving triangles, etc.	5
	Complex	The operation of complex numbers	3
	Solid geometry	Solid geometry	3
	Space vector	Space vector	9
	Plane analytic geometry	Plane analytic geometry	5
Probability and statistics	Probability	Random events, Independence of random events, etc.	11
	Counting	Counting	2
	Statistics	Statistical chart, etc.	12
Calculus	Derivative of a univariate function and its applications	The concept of derivative and its significance, etc.	4
	Derivative and differentiate	Derivative and differentiate	6
	Definite integral	Definite integral	2

Data and processing

All coding and data analysis utilize three applications: NVivo20, Python 3.7 and Excel 2019. Due to the fact that many core mathematical vocabulary is identified using formulas or different letters, it is not possible to directly use NVivo for coding matching. Therefore, it is necessary to use manual means to assist in matching and encoding.

Results

The Internal comparison of IB between AA & AI

Comparison Chart of Knowledge Points of AA and AI

C1:Number and algebra
C2:Functions
C3:Geometry and trigonometry
C4:Statistics and probability
C5:Calculus

Figure 2

Comparison Chart of Knowledge Points of AA and AI

It is evident that both the Analysis and Application courses encompass a set of common knowledge points as shown in Figure 2, while also featuring unique elements within each course type. An analysis of the knowledge points across these courses (p=0.62) indicates that while there isn't a stark contrast between the two, there are nonetheless discernible differences, despite the absence of an overtly significant effect.

The specific distinctions are articulated as follows: In the domain of Numbers and Algebra, the Analysis and Approaches (AA) course primarily centers around resolving ternary linear equations, whereas Applications and Interpretation (AI) course delves into matrix operations. Regarding Functions, AA concentrates on transformations of abstract functions, such as translation and scaling, contrastingly, AI investigates various functional models including linear, quadratic, and exponential function models. In the Geometry and Trigonometry section, AA explores three-dimensional space vectors and vector operations, while AI navigates through graph theory, a niche area seldom incorporated in other syllabi. Within Statistics and Probability, AA familiarizes students with an array of probability formulas, such as the Bayes formula, along with mean and correlation coefficient; conversely, AI adopts a more in-depth approach, studying hypothesis testing and employing samples for population estimation. Moreover, AI introduces concepts typically associated with advanced probability theory, such as the Markov chain and state transition matrix. In the Calculus segment, AA elaborates on derivatives, covering aspects like the definition of derivatives, the derivative of an inverse function, L'Hôpital's rule, higher derivatives, and the Maclaurin series; AI, on the other hand, explores solutions to differential equations, including classical numerical solutions like the Euler method.

Examining the curriculum content, it is manifest that AA and AI present composite curriculum standards, each with distinct orientation and objectives. AA acknowledges the necessity for analytical acumen in a world where innovation is progressively reliant on an in-depth comprehension of mathematics. In contrast, AI identifies the escalating significance of mathematics and technology across diverse fields in a data-rich environment (IB Diploma Programme, 2019).

The Internal comparison of MC between Class A & Class B

Both the Class A and Class B courses serve as elective options. In contrast, both the core and the selectively mandatory sections are requisite for all students, thus eliminating the need for redundancy in our comparative analysis.

Utilizing a consistent methodology, our findings are depicted in Figure 3 below:
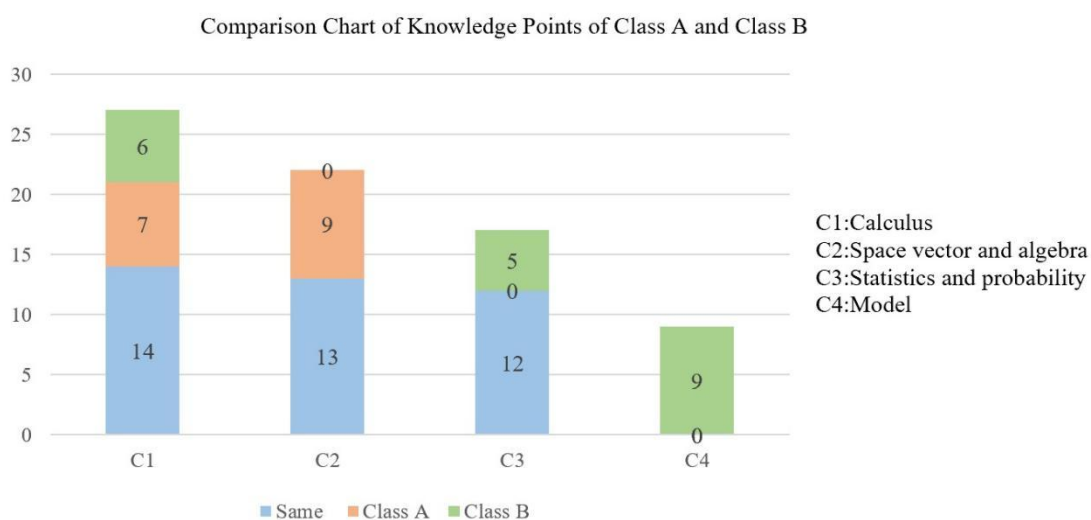
Comparison Chart of Knowledge Points of Class A and Class B

C1:Calculus
C2:Space vector and algebra
C3:Statistics and probability
C4:Model

Figure 3

Comparison Chart of Knowledge Points of Class A and Class B

Firstly, it's imperative to recognize that within the MC curriculum, a substantial portion comprises mandatory core and optional core content. Consequently, there's a limited array of mathematical knowledge points present in the elective segment. As discerned from the aforementioned figure, there's a minimal overlap in knowledge points between Class A and Class B courses, restricting them to merely four discernible categories.

In the realm of calculus, Class A delves into the intricacies of limits, exploring functional limits and emphasizing two pivotal limit concepts. Conversely, Class B places a premium on exploring the zeniths and nadirs of binary functions. Within the scope of spatial vectors, Class A provides a comprehensive analysis, addressing both the linear equation systems and spatial vectors in parallel with Class B, but it further embarks on a journey through plane transformations, interlacing analytic geometry with planar geometry. Significantly, Class A melds this content with modern algebra, a curriculum predominantly reserved for undergraduate mathematics majors in numerous countries. In the statistical domain, Class B ventures deeper into cluster analysis, orthogonal design, and bi-linear regression, whereas Class A's statistical content is comparatively restrained. As for modeling, Class A abstains entirely. This thematic content is the hallmark of Class B, encapsulating linear, quadratic, exponential, and trigonometric function models.

A comparative lens on the dichotomy between Class A and Class B elucidates distinct pedagogical leanings: Class A is rooted in theoretical exploration, mirroring the AA courses of IB, while Class B pivots towards practical mathematical applications, reminiscent of the AI courses.

The comparison between IB and MC

From the intra-comparison of IB and MC courses discussed in sections 3.1 and 3.2, it becomes evident that AA courses align closely with the content of Class A courses, while AI courses and Class B courses share parallel guiding principles. To elucidate the disparities between them, we employ the identical methodology previously used for juxtaposing IB and MC.
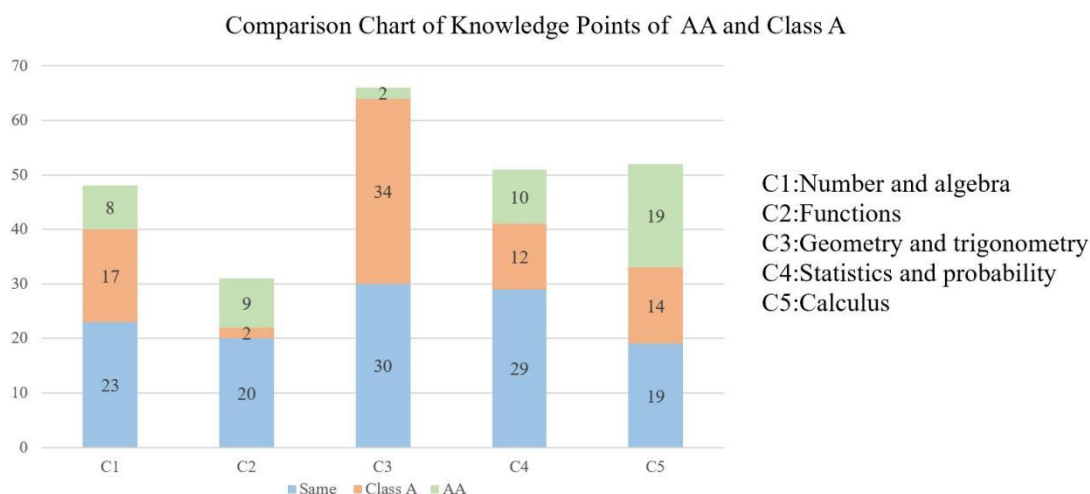
The Comparison between AA and Class A

Comparison Chart of Knowledge Points of AA and Class A

C1:Number and algebra
C2:Functions
C3:Geometry and trigonometry
C4:Statistics and probability
C5:Calculus

Figure 4

Comparison chart of knowledge points of AA and Class A

However, through Figure 4, we can see a different landscape. In the calculus part, Class A also have almost half of their knowledge points AA is not involved. It can be seen that there are obvious differences between the two course standards in this part. AA studies more indefinite integrals, differential equations, and Maclaurin series, which belong to new applications. However, Class A has studied more about sequence limits and the Lagrange theorem, which belong to the content of proof. In geometry and trigonometry, the difference between the two curricula is even greater. Class A also has its own advantages, and almost half of the contents AA are not covered. The main reason is that Class A courses teach:

(1) geometric proof, such as the determination of the relationship between plane and line, etc.

(2) in-depth exploration of circle, hyperbola, ellipse, parabola, etc. in analytic geometry.

Under the topic of Number and algebra, Class A explains sets, basic logic languages, inequalities, etc. AA explained geometric series, and the De Morgan theorem, and introduced the method of contradiction, which is not available in Class A. In Statistics and probability, Class A focuses on the definition of normal distribution, while AA courses focus on the use of normal distribution for calculation. And Class A involves two-dimensional normal distribution, binary linear regression model, etc.

On the whole, although it is affirmed in the previous article that AA and CA are highly similar in general, carefully comparing the details, AA courses still focus more on application and CA focuses more on the understanding of definitions. And it can be seen that the number of learning knowledge points in Class A courses is significantly more than that of AA.

Upon examining Figure 4, a contrasting picture emerges. Within the realm of calculus, nearly half of the topics covered in Class A are absent in the AA curriculum, highlighting a discernible distinction between the two course standards. While AA delves deeper into areas such as indefinite integrals, differential equations, and the Maclaurin series, showcasing newer applications, Class A leans more towards foundational concepts like sequence limits and the Lagrange theorem, emphasizing proof-driven content.

The disparities become even more pronounced in the geometry and trigonometry sections. Class A has a distinct edge, addressing nearly half of the content untouched by AA. This distinction primarily arises from Class A's emphasis on:

(1) Geometric proofs, including determinations of the relationships between planes and lines.

(2) A comprehensive exploration of conic sections like circles, hyperbolas, ellipses, and parabolas within analytic geometry.

Within the Number and Algebra category, Class A delves into sets, foundational logic terminology, and inequalities. Conversely, AA touches upon the geometric series and the De Morgan theorem, introducing the method of contradiction, an element not found in Class A. In the realm of Statistics and Probability, Class A zeroes in on the definition of normal distribution, while AA places emphasis on its practical applications. Furthermore, Class A incorporates topics like two-dimensional normal distribution and binary linear regression models.

In summation, while our previous discussion suggested a high degree of congruence between AA and Class A, a detailed analysis reveals nuances. AA courses appear to have a more application-oriented focus, whereas Class A emphasizes a deeper understanding of core definitions. Additionally, it's evident that Class A offers a more expansive range of topics compared to AA.

The Comparison between AI and Class B

Comparison Chart of Knowledge Points of AI and Class B

C1: Number and algebra
C2: Functions
C3: Geometry and trigonometry
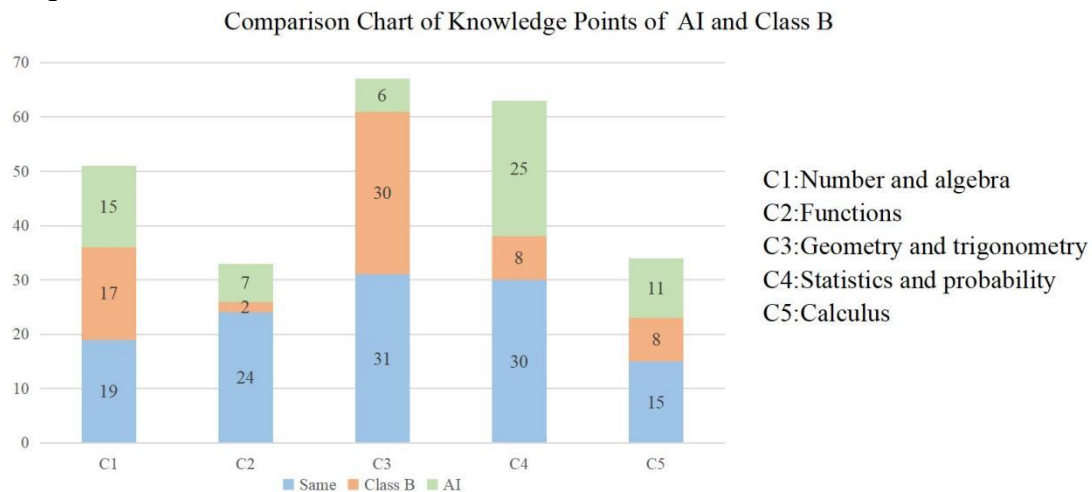C4: Statistics and probability
C5: Calculus

Figure 5

Comparison chart of knowledge points of AI and Class B

In assessing the content overlap between AI and Class B, the distinction is relatively minor, particularly in terms of specific knowledge points. Within the realm of numbers and algebra, the disparity parallels the one observed between AA and Class A. However, this divergence is largely quantitative in nature rather than qualitative. Notably, the AI course delves into the utilization of scientific calculators for matrix operations and eigenvalue computation—topics traditionally reserved for university-level linear algebra. The unique content in Class B mirrors that of Class A, with the discrepancies rooted in the compulsory components of the MC curriculum.

Regarding the function topic, the congruence between the two courses is striking. Both elucidate on functional traits and models, encompassing quadratic and exponential functions, underscoring a harmonized approach in this domain.

Contrast becomes evident in the geometry and trigonometry sections. The bulk of the variance in the Class B outline stems from the MC curriculum's compulsory segment, which emphasizes geometric proofs. A smaller portion of this difference arises from the optional

compulsory content—specifically, plane analytic geometry. This divergence mirrors the previously noted contrast between AA and Class A. Within the statistics and probability topic, AI exhibits a pronounced edge, integrating advanced concepts from university-level probability theory and mathematical statistics, such as the central limit theorem and the estimation of population distributions using sample data.

However, it would be premature to conclude that the MC curriculum is inherently deficient in statistics. The MC's Class C is tailored for students with a predilection for the humanities and social sciences, and it includes a substantial amount of mathematical statistics. Yet, this content is absent from our comparison due to foundational assumptions we've made.

When examining the calculus subject, AI and Class B showcase divergent priorities. AI accentuates integral solutions and numerical approaches to differential equations, whereas Class B leans towards binary function optimization.

To summarize, while Class B encompasses a marginally larger set of knowledge points compared to AI, the volume difference observed between AA and Class A is less pronounced.

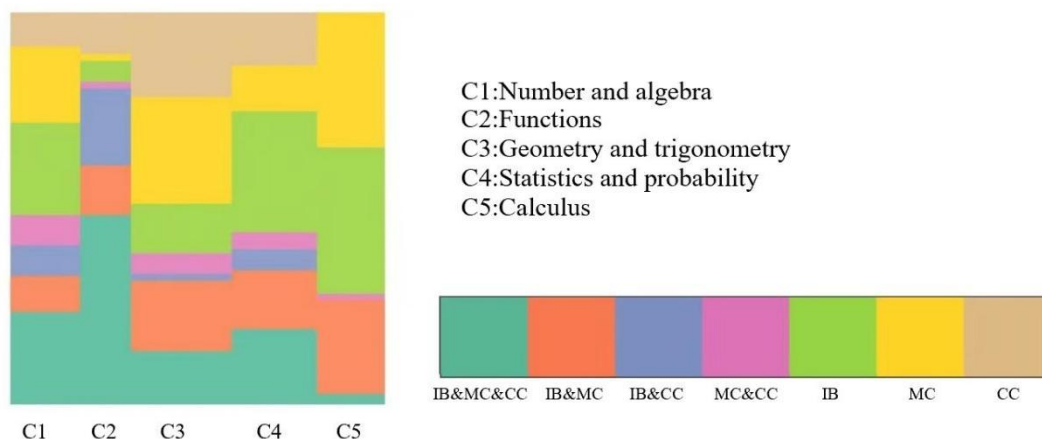**The comparison among IB and MC and CC**

C1:Number and algebra
C2:Functions
C3:Geometry and trigonometry
C4:Statistics and probability
C5:Calculus

Figure 6
Comparison of knowledge points of IB and MC and CC

To illuminate the disparities among the top three curriculums, we transitioned from the previously employed stacked histogram to proportion staining in Python. This approach enables an examination of each topic's representation within the broader curriculum via the width of the abscissa, while the ordinate reveals the proportional distribution of knowledge points under each topic within each curriculum in Figure 6.

A striking observation is the minimal overlap among the three. This suggests a lack of consensus on the essential mathematical competencies expected at the high school level. Among all topics, the domain of functions demonstrates the most alignment across the trio – an unsurprising observation given its undisputed foundational role in modern mathematics.
The topic of geometry and trigonometry exhibits the most substantial divergence and also claims the lion's share of knowledge points. This is predominantly because MC emphasizes proofs, delving deeper into spatial properties of lines and planes. In contrast, IB introduces graph theory, with its primary focus on planar geometry aspects like congruence and symmetry in triangles.

Regarding numbers and algebra, the curriculums appear to be charting distinct paths. While all three address exponents, logarithms, complex numbers, and sequences, they diverge significantly in other areas: IB explores matrix power operations, MC elucidates sets, logic, and inequalities, and CC ventures into polynomial theory.

In the realm of statistics and probability, CC stands out despite the absence of AP courses,

reflecting the significant emphasis the U.S. places on this domain. While MC has its unique content, IB particularly shines due to its AI segment, which integrates comprehensive statistical concepts such as the central limit theorem, sample population estimation, and Markov chains, granting IB a discernible edge in this area.

When examining calculus, clear distinctions emerge between IB and MC. As previously noted, IB leans towards problem-solving, whereas MC prioritizes foundational definitions.

Incorporating AP courses would considerably amplify CC's robustness. In statistics and probability, it virtually encapsulates all the knowledge points addressed by IB and MC, delving further into topics like the law of large numbers and dual population mean tests. An informal coding revealed 16 unaddressed directions of knowledge points. Similarly, in calculus, CC surpasses others significantly. It thoroughly covers advanced topics, from indefinite integrals to differential equations and series, outpacing the more rudimentary concepts presented in IB and MC. An astounding 70 knowledge point directions remained untouched in our coding.
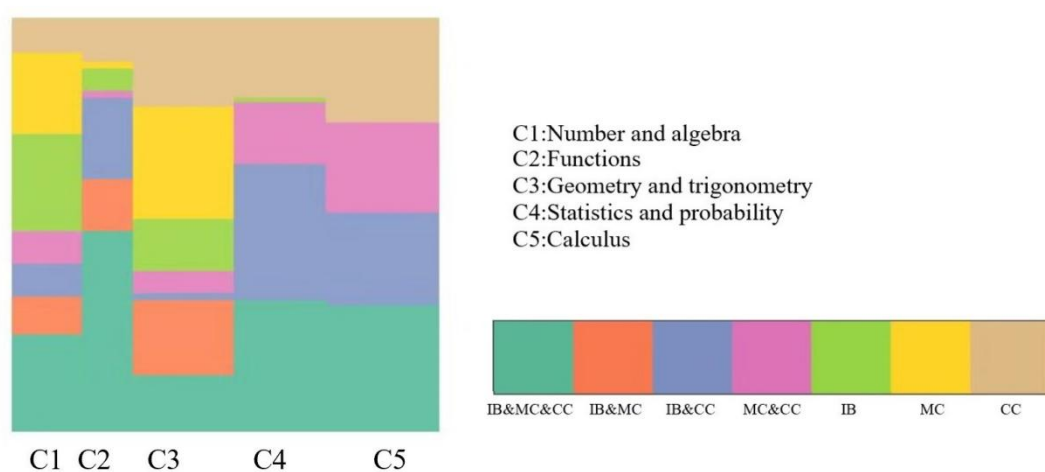

C1:Number and algebra
C2:Functions
C3:Geometry and trigonometry
C4:Statistics and probability
C5:Calculus

Figure 7

Comparison of knowledge points of IB and MC and CC&AP

Considering the count of knowledge points and excluding the impact of AP courses in Figure 7, MC encompasses the broadest range, followed by IB, while CC has the fewest. However, when incorporating AP courses into the analysis, the disparities in the number of knowledge points across the three become marginal. In this context, CC slightly edges out with a few additional points, followed closely by MC, leaving IB with the fewest.

The Comparison in mathematics guide

Given the considerable disparities across the IB, MC, and CC in various aspects, is there a significant divergence among these three course types in the overarching domain of mathematics?

To obtain a comprehensive perspective on the mathematical concepts covered in these three curriculums, we juxtaposed them with classifications found in the "Mathematical Guide Practical Mathematics Manual" (by Eberhard Zeidler, 2012), authored by the renowned German mathematician Eberhard Lederer.

The principles of comparison are delineated as follows: The mathematical guide is structured into eight primary chapters, each further segmented into specific topics. Within each topic, multiple concepts are detailed. For a precise representation of the concepts covered by each curriculum type, they will be cataloged at the granular level of individual concepts within broader chapters. For instance, the concept "Real Numbers" within the first chapter on analysis will be marked if it is mentioned in any of the course standards. Owing to the vast scope of mathematical content, this section will log a concept as long as it is mentioned at least once within the
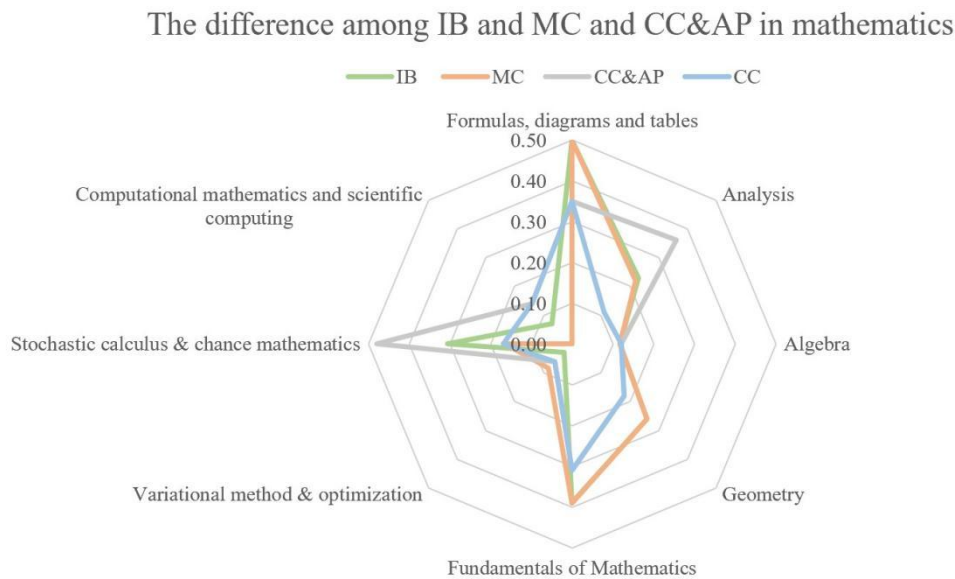
curriculums.

Figure 8

The difference among IB, MC and CC&AP in mathematics

From the analysis, it is evident that IB, MC, and CC&AP touch upon only a limited spectrum of mathematical content within diverse branches of the discipline. The primary focus of these curriculums is on fundamental formulas, graphical representations, and foundational mathematical principles, with a modest inclusion of analytical mathematics, algebraic geometry, and probability theory. Surprisingly, variational methods and optimization, which form the bedrock of Applied Mathematics, find scant mention across these curricula. Given that the revised curriculum standards have omitted content pertaining to algorithms, areas such as computational mathematics and scientific computing remain absent in typical high school learning.

Discussion

When contrasting the syllabi of IB, MC, and CC, distinct variances in the treatment of mathematical knowledge points emerge, though they all preserve core mathematical content. In the domain of number and algebra, there is a consensus on the study of number sequences, generally perceived as discrete functions. This emphasis is pivotal for comprehending theory within the scope of integers. The exploration of complex numbers has also deepened, likely driven by the significance of Euler's formula and the crucial role of complex numbers in expanding the numerical system.

Regarding the subject of functions, there's unanimity that core functions like exponential, logarithmic, quadratic, polynomial, and trigonometric need thorough coverage. These functions form the bedrock of elementary functions, vital not just for calculus but also for real-world applications.

The theme of geometry and trigonometry presents divergence. While IB has an application-centric approach emphasizing graph theory, MC and CC prioritize training and proofs. MC delves into theories of straight lines and planes, while CC immerses students in plane geometry's triangle theories, aiming to instill a comprehensive logical reasoning system.

In the realm of statistics and probability, superficial differences mask an underlying similarity. As demonstrated by the proportion of mathematical knowledge points in the three curricula, they largely traverse the same terrain, with variations in content focus. This is even more palpable in calculus; despite any advancements from AP courses, students' exposure remains largely computational, not truly capturing the essence of analysis.

The trajectories of computational mathematics and scientific computation, including variational methods and optimization, remain underrepresented across all three syllabuses. According to the latest PISA guidelines (OECD, 2021), the proficiency in computational tools and technology usage is emphasized. While IB (both AA and AI) mandates technology utilization, such as using calculators for inverse matrices or plotting functions, and CC explicitly promotes electronic tabulation software or computer algebra systems, MC's stance remains nebulous. This ambiguity risks hampering the accurate execution of curriculum standards. Efforts in China, like introducing artificial intelligence courses (China Education Society, 2021), attempt to bridge this gap. However, disjointedness between this content and core mathematics persists. CC might have an edge in technological application, but proficiency in tools doesn't necessarily equip students to address more advanced challenges, like solutions for high-order differential equations or multivariate function optimization. This might explain the sparse coverage of scientific computation, variational methods, and optimization in these syllabuses.

The ongoing debate over curriculum content, be it the "new number movement" or the "return to basics", remains a salient discourse in both domestic and international spheres (Cheng & Bao, 2019).

Conclusion

(1) In the education of gifted students, excluding the influence of AP courses, the MC curriculum encompasses the broadest range of mathematical knowledge points. IB closely parallels MC in its scope, while CC offers the fewest knowledge points. However, when we factor in AP courses, the distinction in knowledge points among the three curricula diminishes significantly. This challenges the prevalent perception among many American scholars that their curriculum standards operate on a "mile-wide, inch-deep" principle. Conversely, it also counters the Chinese academic viewpoint that categorizes the curriculum as "inch-wide, mile-deep".

(2) When it comes to technological integration, MC offers a generic mention of science and technology utilization. In contrast, IB specifically advocates for the use of scientific calculators, and CC emphasizes computer applications. This vague stance in MC potentially renders its technology application nebulous and less actionable in an educational setting.

(3) Across the entire spectrum of mathematical instruction, the current technological provisions seem insufficient to address advanced computational problems. This is evident from the limited emphasis on computational mathematics, scientific computation, variational methods, and optimization across all curricula.

(4) Upon including the content from AP courses, it becomes apparent that the mathematical knowledge encompassed in the senior high school curricula of IB, MC, and CC&AP is quite limited within the vast field of mathematics. Notably, CC&AP emerges as the most comprehensive among the three, albeit with a slight edge.

Limitations and Future Work

This study focuses on the enumeration of knowledge points without delving into the complexity of problems stemming from these points, an aspect better suited for empirical research with a measurement-based approach. Furthermore, while the research sheds light on the mathematical content available for gifted students within the curriculum, it does not address the advanced mathematical content encountered in competitions like the International Mathematical Olympiad (IMO). Additionally, China's initiative to bolster its educational infrastructure remains outside the scope of this analysis, but it could be an interesting avenue for future studies.

Comparative analysis does not necessarily render a definitive verdict on the superiority or rationality of one curriculum over another. Given the constraints of academic scheduling, it is ambitious to expect comprehensive coverage of the vast domain of mathematics within a restricted timeframe. Consequently, each curriculum, in its design and emphasis, reflects specific considerations and orientations.

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LX Contributed to the conception and design of the study. LX and JY collected data and encoded data. LX and JY drafted sections of the manuscript. LX and CP did the data analysis and drew the plots. XB secured funding for the study. LX and XB rewrote and revised the manuscript. All authors contributed to the article and approved the submitted version.

Ethics statement
This study is a document analysis that does not involve ethical review.